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OPTIMUM FORMS OF THREE-DIMENSIONAL BODIES FOR PENETRATION OF DENSE MEDIA

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This article reports on the results of a study of the optimum form of three-dimensional bodies for the penetration of dense media in cases in which, given certain assumptions, the interaction of the medium and the body can be examined within the framework of the law of locality [1]. The method of local variations [2] was used to develop a numerical algorithm to search for forms of the body that would maximize the depth of penetration of the medium.

Examples are presented of the solution of a variational problem with different isoperimetric conditions on the geometry of the body. The examples show that, in terms of depth of penetration, three-dimensional bodies whose form has been optimized may have a significant advantage over equivalent traditional solids of revolution.

1. Classes of Bodies. We will examine the motion of a body whose form is described by the following equation in a cylindrical coordinate system (r, θ, x), with its origin at the tip of the body and the x-axis directed oppositely to the direction of motion

$$f(r, \theta, x) \equiv r - \varphi(x)R(\theta) = 0, \tag{1.1}$$

where $\varphi(x)$, $R(\theta)$ are functions determining the longitudinal and transverse contours of the body, respectively. Here, $\varphi(0) = 0$, $\varphi(L) = 1$ (where L is the specified characteristic length of the head of the body).

We will henceforth assume that the longitudinal contour of the head is known and is given by the equation

$$\varphi(x) = x/L = \xi. \tag{1.2}$$

If the head, the area of the center section of which is S_m, is a cone of length L, then $R(\theta) = \sqrt{S_m/\pi} = L\tau/2$ (τ is the relative thickness of the head) and $\varphi(x) \equiv 1$ at $x > L$.

If we want to shape the head so as to maximize depth of penetration for a given impact velocity v_0 and cross-sectional area S_m, we need to consider that different isoperimetric conditions may be imposed on the body, depending on the practical requirements. The conditions most often encountered are specification of the mass (volume) of the body and limitation of its transverse dimensions.

There are two basic approaches to resolution of the problem.

In the first approach, depth of penetration is increased by the "deformation" of a certain frontal region of the body (Fig. 1). This region has a circular midsection and a head with a specified relative thickness τ (dashed lines). In this case, the surface of the new, conical head will contact with the circular cylinder along a space curve

$$\xi g(\theta) - \tau/2 = 0, \; g(\theta) = R(\theta)/L. \tag{1.3}$$

Given a known function $g(\theta) \not\equiv \tau/2$, the head will be conical for $\xi = \xi_1 = \tau/(2g_1)$ (see Fig. 1) and will consist of conical and cylindrical surfaces with $\xi_1 < \xi < \xi_2 = \tau/(2g_0)$, where g_1 and g_0 are respectively the largest and smallest values of $g(\theta)$ on the segment $[0, 2\pi]$.

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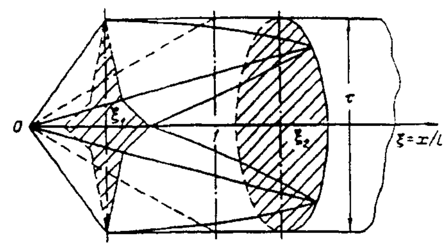

Fig. 1

As does the equivalent comparison body (circular cone — cylinder), bodies of the type just described have a midsection in the form of a circle and will be referred to as bodies of class 1.

The second approach to optimizing the shape of the body involves making a transition from an equivalent solid of revolution with the transverse dimension $2\sqrt{S_m/\pi}$ to a three-dimensional body with a large transverse dimension and the same midsection area S_m. In this case, we are optimizing the head of the body over the length L ($\xi \leq 1$), while at $\xi > 1$ we have a cylindrical surface stretched over the contour of the midsection of the three-dimensional head (Fig. 2). We will refer to bodies of this type as class 2 bodies.

The above two classes of bodies suggest the types of configurations that might maximize depth of penetration compared to equivalent solids of revolution. They also determine the types of variational problems that should be examined. We will discuss these problems below. It should be noted that solids of revolution with a conical head having a relative thickness τ are among the shapes included in classes 1 and 2.

2. Depth of Penetration of the Body. Let S be the surface on which the body makes contact with the medium. Then the resistance of the body can be written in the form

$$D = q\iint_S [-c_p(\mathbf{n}\cdot\mathbf{x}) + c_\tau(\tau\cdot\mathbf{x})]dS, \tag{2.1}$$

where $q = \rho v_0^2/2$ (ρ is the density of the medium); $\mathbf{x}$, $\mathbf{n}$, and τ are unit vectors on the x-axis, an outer normal to the surface of the body, and a tangent to the surface directed downflow at the point being examined; here, $\tau[\mathbf{n} \times \mathbf{x}] = 0$, which in most cases is the only assumption made; c_p and c_τ are coefficients expressing the pressure and friction on the surface of the body.

Within the framework of the law of local interaction — when it is assumed that the force exerted by a medium on an element of surface area of a body depends only on its orientation relative to the direction of motion — the coefficients c_p and c_τ can be represented by the expressions

$$\begin{aligned} c_p &= A_1(v/v_0)^2(\mathbf{n}\cdot\mathbf{x})^2 - B_1(v/v_0)(\mathbf{n}\cdot\mathbf{x}) + C_1, \\ c_\tau &= A_2(v/v_0)^2(\mathbf{n}\cdot\mathbf{x})^2 - B_2(v/v_0)(\mathbf{n}\cdot\mathbf{x}) + C_2. \end{aligned} \tag{2.2}$$

In the general case, coefficients A_i, B_i, C_i ($i = 1, 2$) of local model (2.2) depend on the characteristics of the medium and impact velocity v_0. Either they can be empirical constants [5, 6] or, with certain assumptions, they can be determined from the theory in [3, 4].

We will restrict ourselves to the case when c_p and c_τ are connected by the relation

$$c_\tau = \mu c_p \tag{2.3}$$

(μ is the coefficient of Coulomb friction). We further assume that we can ignore the decrease in impact velocity v_0 over the period of time during which the head of the body has not yet fully penetrated the medium. Estimates have shown this to be valid for impact velocities $v_0 \gtrsim 10(\tau/2)^{-1}$ m/sec and $v_0 \gtrsim (10^2\text{-}10^{5/2})(\tau/2)^{-1}$ m/sec when the impact is against the ground and metallic barriers, respectively. Then using Eqs. (2.1-2.3) and integrating the equation of motion of a body of mass M in the case of an impact which is normal to the surface of the medium, we obtain the following expression for the depth of penetration:

$$H = \frac{Mv_0^2}{2qAY_1}\left\{\ln\left(1 + \frac{B}{C}\frac{Y_2}{Y_3} + \frac{A}{C}\frac{Y_1}{Y_3}\right) + \Phi\right\}. \tag{2.4}$$

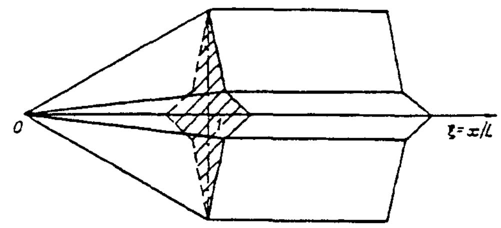

Fig. 2

The subscript 1 was omitted from A, B, and C in (2.4); the quantity Φ has different analytic representations, depending on the value of $E = 4ACY_1Y_3/(BY_2)^2 - 1$:

1) when
$$E > 0$$
$$\Phi = 2E^{-1/2}[\mathrm{arctg}(E^{-1/2}) - \mathrm{arctg}(E_0 E^{-1/2})],$$
$$E_0 = 1 + 2AY_1/(BY_2);$$

2) when
$$E = 0$$
$$\Phi = -2(1 - E_0^{-1});$$

3) when
$$E < 0$$
$$\Phi = (-E)^{-1/2}\ln\left[\frac{(E_0 + \sqrt{-E})(1 - \sqrt{-E})}{(E_0 - \sqrt{-E})(1 + \sqrt{-E})}\right].$$

When B = 0 (which corresponds in particular to the Sagomonyan models for soils and metals [3, 4] and the Zabudskii model for soils [3, 5]), Φ vanishes and the expression for depth of penetration takes the form

$$H = \frac{Mv_0^2}{2qAY_1}\ln\left(1 + \frac{A}{C}\frac{Y_1}{Y_3}\right). \tag{2.5}$$

When A = C = 0 (Berezanskaya model for soils [5]), the functional H can be written in the form

$$H = \frac{Mv_0^2}{qBY_2}. \tag{2.6}$$

In Eqs. (2.4-2.6), Y_i ($i = 1, 2, 3$) are functionals dependent both on the form of the surface S over which the body makes contact with the medium and on the friction coefficient μ:

$$\begin{aligned} Y_1 &= \iint_S [-(\mathbf{n}\cdot\mathbf{x}) + \mu(\tau\cdot\mathbf{x})](\mathbf{n}\cdot\mathbf{x})^2 dS, \\ Y_2 &= -\iint_S [-(\mathbf{n}\cdot\mathbf{x}) + \mu(\tau\cdot\mathbf{x})](\mathbf{n}\cdot\mathbf{x}) dS, \\ Y_3 &= \iint_S [-(\mathbf{n}\cdot\mathbf{x}) + \mu(\tau\cdot\mathbf{x})] dS. \end{aligned} \tag{2.7}$$

Using a cavitational scheme to characterize flow about the body, we take S to be the surface of the head.

In the general case, the problem of determining maximum depth of penetration reduces to a variational problem which involves finding the maximum of functional (2.4) in a class of surfaces satisfying prescribed isoperimetric conditions. For bodies whose surfaces belong to classes 1 and 2, we write integrals (2.7) as follows in accordance with (1.1)-(1.3)

$$\begin{aligned} Y_1 &= \frac{L^2}{2}\int_0^{2\pi}\left[z\frac{g^3\Phi_2}{(\Phi_1 + g^2)}\right]d\theta, \\ Y_2 &= \frac{L^2}{2}\int_0^{2\pi}\left[z\frac{g^2\Phi_2}{(\Phi_1 + g^2)^{1/2}}\right]d\theta, \\ Y_3 &= \frac{L^2}{2}\int_0^{2\pi}[zg\Phi_2]d\theta, \\ \Phi_1 &= 1 + (\dot{g}/g)^2,\ \Phi_2 = g + \mu\Phi_1^{1/2}. \end{aligned} \tag{2.8}$$

In (2.8), $\dot{g} = dg/d\theta$; z is the parameter characterizing the class of bodies, $z = (\tau/(2g))^2$ for bodies of class 1 and $z = 1$ for bodies of class 2.

3. Formulation of the Variational Problem. Functionals (2.4)-(2.6) are not standard linear functionals and are difficult to study analytically. For thin bodies ($g^2 << 1$), an analytic solution to the variational problem can be obtained by using the method described in [1]. The transverse contour of the optimum thin body consists of n identical cycles (see Figs. 1 and 2, n = 4), each of which contains two symmetric arcs with $\dot{g} \geq 0$ and $\dot{g} \leq 0$, respectively. Such bodies are referred to as radial in aerodynamics.

For non-thin bodies, it is not possible to analytically solve the variational problem of maximizing functional (2.4) for different isoperimetric conditions.

Proceeding on the basis of results of studies of optimum thin bodies [1], we will assume that the heads of bodies belonging to classes 1 and 2 are of cyclically symmetric form. Then the search for the optimum surface of the head reduces to searching for $g(\theta)$ in a half-cycle of the transverse contour (with n given) under the condition

$$\dot{g}(\theta) \geq 0, \theta \in [0, \pi/n]. \tag{3.1}$$

The number of cycles n is chosen either on the basis of design considerations or by comparing the depth of penetrations of optimum bodies with different numbers of cycles.

Equation (2.8) has the following form for cyclically symmetric surfaces

$$\begin{aligned} Y_1 &= L^2 n \int_0^{\pi/n} \left[z \frac{g^3 \Phi_2}{(\Phi_1 + \dot{g}^2)} \right] d\theta, \\ Y_2 &= L^2 n \int_0^{\pi/n} \left[z \frac{g^2 \Phi_2}{(\Phi_1 + \dot{g}^2)^{1/2}} \right] d\theta, \quad Y_3 = L^2 n \int_0^{\pi/n} [z g \Phi_2] d\theta. \end{aligned} \tag{3.2}$$

We examined the following variational problems for bodies of class 1 (see Fig. 1) to determine the optimum form for the head. The problems differ in the isoperimetric and boundary conditions imposed on the transverse contour $g(\theta)$.

1. "Deformation" of the head takes place with conservation of its volume, which corresponds to the condition

$$\int_0^{\pi/n} \frac{d\theta}{g} = \frac{2\pi}{\tau n}. \tag{3.3}$$

In this case, no limitations are imposed on the ends of the extremal $g(\theta)$.

2. The volume of the body is conserved (3.3) and the minimum dimension of the transverse contour is prescribed ($g_0 \equiv g(0) < \tau/2$).

3. The volume of the body is not conserved, but the ends of the extremal are fixed in the section $\xi = 1$:

$$g_0 < \tau/2, \; g_1 \equiv g(\pi/n) = \tau/$$

In problems 1 and 2, $\xi_1 \leq 1$, $\xi_2 = \tau/(2g_0)$ (see Fig. 1) because of volume conservation condition (3.3). In problem (3), the radial and cylindrical surfaces come into contact at $\xi_1 = 1$.

For bodies of class 2 (Fig. 2), we examined two problems in addition to variational problems 1-3. In the case of bodies of class 2, the volume conservation condition — which here coincides with the condition of conservation of the area of the midsection of the body — takes the form

$$\int_0^{\pi/n} g^2 d\theta = \frac{\pi}{n} \left(\frac{\tau}{2} \right)^2, \tag{3.4}$$

The two additional problems that were examined are as follows.

4. The volume of the body and the maximum radius of the transverse contour are both assigned ($g_1 > \tau/2$).

5. The volume and the maximum g_1 and minimum g_0 radii of the transverse contour of the body are all assigned.

Here, the functional $\bar{H}$ is maximized. This functional is equal to the ratio of H (2.4) to the corresponding depth of penetration of an equivalent solid of revolution having a conical head with a relative thickness τ. This approach clearly shows the advantages of optimum three-dimensional bodies of classes 1 and 2 compared to an equivalent solid of revolution. In addition, $\bar{H}$ is independent of the mass of the body.

In accordance with (2.4), (3.2)-(3.4), if no conditions are imposed on the ends of the extremal, the solution of the problem will depend on the following parameters:

$$\frac{\tau}{2}, \frac{A}{C}\left(\frac{\tau}{2}\right)^2, \frac{B}{C}\left(\frac{\tau}{2}\right), \frac{2\mu}{\tau}, n. \tag{3.5}$$

The order of v_0 at which we can ignore the loss of velocity during the period in which the head has not yet fully penetrated the medium corresponds to cases in which the second parameter in (3.5) has a value $\gtrsim$ O(1). If the ends of the extremal are fixed, then $2g_0/\tau$ and $2g_1/\tau$ must also be added to (3.5). The first parameter will be absent from (3.5) for thin bodies ($(\tau/2)^2 << 1$). The fourth parameter in (3.5) indicates that Coulomb friction is an important parameter of the problem, since $2\mu/\tau$ = O(1) for actual media. It should also be pointed out that it is not possible to obtain additional information on limiting the number of parameters determining the solution of the problem without using necessary conditions for the extremum. This was done in [1] for thin bodies, where one condition was a criterion for a change in the optimum contour from circular to noncircular.

4. Method of Solution. To solve the problems formulated above, we developed a numerical algorithm based on the method of local variations [2].

We divide the interval of integration [0, π/n] into N equal parts with the step $\tau_1 = \pi/(nN)$, and we introduce the notation $\theta_m = m\tau_1$ (m = 0,..., N) and $g_m = g(\theta_m)$. Then in accordance with the linear interpolation of $g(\theta)$ and the first order of the finite-difference approximation of its derivative on the interval $\Delta\theta$, the integrals in (2.4) are represented by the formulas

$$\begin{gathered} I_i \equiv Y_i/(L^2n) = \sum_{m=1}^{N} I_{im} \quad (i = 1, 2, 3), \\ I_{im}(g, \dot{g}) = \tau_1 f_i\left(\frac{g_m + g_{m-1}}{2}, \frac{g_m - g_{m-1}}{\tau_1}\right), \end{gathered} \tag{4.1}$$

where f_i are integrands.

In accordance with (4.1), we need to find a distribution of g_m (m = 0,..., N) that satisfies the boundary conditions, condition (3.1), and isoperimetric condition (3.3) (or (3.4)), giving us the maximum of the functional $\bar{H}$.

The algorithm consists of the following steps. We choose the initial approximation for the transverse contour $g(\theta)$ so as to satisfy the isoperimetric and boundary conditions and we calculate $\bar{H}$ on it with the use of (4.1). Then we successively vary the component g_m for each m $\in$ [0, N], where δg_m = h (h being an assigned value). We check the boundary conditions and condition (3.1) for each variation. An additional variation of the contour $g(\theta)$ is performed at a certain point θ_l ($l \neq$ m) to satisfy isoperimetric condition (3.3). Here, the variation $\delta g_l = h_l$ is connected with h for isoperimetric conditions (3.3) and (3.4), respectively

$$h_l = -hg_l^2/g_m^2, \; h_l = -hg_m/g_l. \tag{4.2}$$

After replacing g_m by g_m + h and replacing g_l by $g_l + h_l$, we calculate integrals (4.1) and the functional $\bar{H}$. Of the variations $\delta g_l = h_l$ ($l \neq$ m), we choose that which for the given variation δg_m = h leads to the maximum increase in $\bar{H}$. The procedure just described is repeated for the variation δg_m = −h. As a result, $g(\theta)$ is matched with the new values $g_m + \delta g_m$ that, together with the reciprocal variation of the contour at point θ_l, yield the greatest increment in $\bar{H}$.

The iteration is considered complete if the possible variations of the contour have been completed at all points θ_m. At the end of the iteration, we obtain a new approximation for the distribution of g_m that satisfies the isoperimetric and boundary conditions. The functional $\bar{H}$ for the new approximation is no smaller than for the initial approximation. If $\bar{H}$ does not increase after the next iteration is performed for the assigned h, then h is halved and calculations are continued until δg_m becomes sufficiently small. We then halve τ_1, i.e. we double N — the number of subdivisions of the interval [0, π/n]. The values of g_m at the new points are determined by linear interpolation over adjacent points of the previous distribution and the iteration is begun again. When τ_1 becomes sufficiently small and no significant increase occurs in the functional ($0 < \Delta\bar{H} < \varepsilon$), the computation performed to find the optimum distribution of g_m can be considered complete.

To make clearer the meaning of "sufficiently small," we need to examine the convergence of the above-described method. It can be shown that the solution obtained by the method of local variations satisfies a finite-difference approximation of the Euler equation in the variational problem to within terms of order max (h, h/τ_1^2) on the arcs of the extremal $\dot{g} > 0$. It follows from this that the sought solution will be obtained from a computation performed with $h << \tau_1^2$ and $\tau_1 \to 0$.

A numerical experiment showed that if the ends on segment $[0, \pi/n]$ are not fixed under conditions (3.3)-(3.4) or if only minimum value g_0 of $g(\theta)$ is assigned, the method of local variations ensures convergence to the same solution for any initial distribution g_m. However, as was shown in [1], the weak variations in the given method do not always yield a solution to the variational problem. In the case of isoperimetric conditions (3.3)-(3.4) and boundary conditions in which either g_1 or g_1 and g_0 are assigned, convergence depends on the initial distribution of g_m because of the order of approximation (4.1) and the weak variations of $\dot{g}(\theta)$, which are in turn a consequence of condition (3.1) and the above-indicated ratio of the orders of h and τ_1.

The choice of initial approximation becomes particularly important in this case. Regardless of the boundary conditions, a successful choice substantially reduces the time spent on searching for a solution to the variational problem by the method of local variations.

It is known [1] that the optimum transverse contour of a thin three-dimensional body consists of a set of circle arcs corresponding to sections of surfaces of circular cones and regular arcs ($\dot{g} > 0$) with variable curvature. The arcs contact one another at points of inflection of the transverse contour. However, two factors make it inexpedient to simply adopt the optimum contour in the theory of thin bodies as the initial approximation. First of all, a great deal of time is required to find this contour. Secondly, the optimum contour for a thick body may be quite different.

In light of this, preference is given to an initial approximation which is a contour consisting of circle arcs ($\dot{g} = 0$) and regular arcs ($g > 0$) — segments of straight lines corresponding to plane sections of the surface of the head of the body. The arbitrary parameters involved in the selection of the class of arcs comprising the initial approximation are determined in accordance with the imposed isoperimetric and boundary conditions. If the number of arbitrary parameters is greater than the number of conditions, remaining parameters are chosen on the basis of which yield the highest value of $\bar{H}$. Numerical calculations showed the effectiveness of taking this approach to selection of the initial approximation of contour in the program.

Determination of the form of the head of a body consisting of sections of a circular cone ($\dot{g} = 0$) and planes ($\dot{g} > 0$) so as to maximize $\bar{H}$ (where allowed by the set of isoperimetric and boundary conditions) is a problem which is important on its own. From a practical viewpoint, such a shape is the simplest possible three-dimensional configuration for the head of a body corresponding to the formulation of the problem. In addition, solution of the problem allows us to directly establish the extent to which this shape differs from the configuration that is optimum with respect to depth of penetration.

5. Results of Calculations. As an example, below we present the results of calculations performed to optimize the head of a body. The calculations consisted of four cycles ($n = 4$). The body penetrated loamy clay with an initial velocity $v_0 = 400$ m/sec. For comparison, we examined a circular cone with the relative thickness $\tau = 2\sqrt{S_m/\pi}/L = 2/3$. We used three interaction modes: Sagomonyan's analytical model [3] and the empirical models of Zubudskii and Berezanskaya [5]. The depth-of-penetration functional has the form (2.5) for the first two models and (2.6) for the third model.

Calculations were performed for both classes 1 and 2. The minimum of $g(\theta)$ on $[0, \pi/n)$ was assigned a single value for all cases: $g_0 = 0.25$. Figure 3 shows the optimum cross section of the head of a class 1 body in one of its cycles. The optimization was done within the framework of the model in [3] with the use of isoperimetric condition (3.3). Solid curves 1-3 correspond to the sections $\xi_1 = 0.08$, $\xi = 1$, $\xi_2 = 1.33$ (see Fig. 1), while the dashed curves correspond to the initial approximation of the transverse contour in the sections $\xi_1 = 0.56$, $\xi = 1$. Here, the relative depth of penetration of a body with an optimum head $\bar{H} = 1.89$, while the initial transverse contour $\bar{H}' = 1.48$.

The same isoperimetric condition was used to find the optimum transverse contours of the head within the framework of the Zabudskii and Berezanskaya models [5]. The calculations showed that the type of model has only a slight effect on the optimum form of the head — which is depicted qualitatively in Fig. 3 for the two indicated models. Curves 1 and 2 correspond to the sections $\xi_1 = 0.08$ and $\xi = 1$ for the Zabudskii model, these results agreeing with model calculations in [3]. The relative depth of penetration of a body with an optimum head $\bar{H} = 2.31$. For the initial approximation of its cross section, $\bar{H}' = 1.64$. Accordingly, curves 1 and 2 for the Berezanskaya model correspond to sections $\xi_1 = 0.07$, $\xi = 1$ and $\bar{H} = 1.38$, $\bar{H}' = 1.2$.

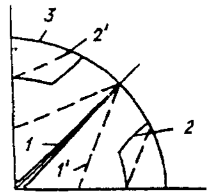

Fig. 3

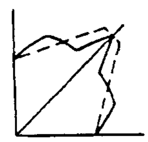

Fig. 4.

Thus, the form of the optimum head is conservative with respect to the choice of the interaction model, given the appropriate isoperimetric and boundary conditions. The difference between the absolute values of $\bar{H}$ and $\bar{H}'$ is due to the different approaches taken to construction of the interaction models that were used and the approximate nature of these models.

Another important result of the calculations was the substantial decrease in the maximum overload p for the body with the optimum head relative to the overload experienced by the solid of revolution with a conical head.

The ratio of the indicated values of $\bar{p}$ (in the case when the head has its initial contour in the cross section $\bar{p}'$) takes the following values for the Sagomonyan, Zabudskii, and Berezanskaya models, respectively: $\bar{p} = 0.42$, $\bar{p}' = 0.62$; $\bar{p} = 0.41$, $\bar{p}' = 0.59$; $\bar{p} = 0.72$, $\bar{p}' = 0.83$.

Figure 4 shows optimum cross sections of class 2 bodies (solid line) and the initial contour (dashed line) in one of its cycles under condition (3.4) and with a fixed maximum of $g(\theta)$ on the interval $[0, \pi/n]$: $g_1 = 0.45$. The results were obtained using the interaction model in [3] for the penetration of loamy clay. The initial contour consisted of straight lines and circle arcs $g(\theta) = g_1$, resulting in natural blunting of the leading edge of the cycle of the head. As in the case of class 2 bodies, the optimum head configuration for class 1 bodies depends slightly on the choice of interaction model. Relative depth of penetration and maximum overload in the given case take the following values for the three above-indicated interaction models: $\bar{H} = 1.29$, $\bar{p} = 0.71$, $\bar{H}' = 1.23$, $\bar{p}' = 0.76$; $\bar{H} = 1.38$, $\bar{p} = 0.71$, $\bar{H}' = 1.3$, $\bar{p}' = 0.76$; $\bar{H} = 1.14$, $\bar{p} = 0.87$, $\bar{H}' = 1.12$, $\bar{p}' = 0.89$.

The Berezanskaya model yields lower values of relative depth of penetration than the Sagomonyan and Zabudskii models for both classes of bodies. The optimum head configurations differ little among these models. The lower values of relative depth of penetration obtained with Berezanskaya model can be attributed to the different analytical representations of the forces acting on the body-medium interface as a function of v. This relationship is linear in the Berezanskaya model and quadratic in the other models. At the same time, although the Sagomonyan and Zabudskii models have different origins, they predict similar depths of penetration.

On the whole, the calculated data shows that there is a significant advantage to be obtained in terms of depth of penetration by changing over from an equivalent solid of revolution to optimum three-dimensional bodies.

In conclusion, we note that, qualitatively speaking, the optimum contours obtained here by numerical means are in complete agreement with the conclusions reached in [1] with the imposition of different boundary conditions on the extremal.

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